Second Czech-Catalan Conference in Mathematics

Barcelona

September 21 to 23, 2006

Welcome

This is the second conference organized jointly by the Czech and Catalan Mathematical Societies.

An agreement was signed in 2004 for reciprocal membership and cooperation between the two societies. This agreement, together with many strong cooperation links between Catalan and Czech mathematicians, made the First Czech-Catalan Conference in Mathematics a reality and a great success in Prague, May 27 and 28, 2005.

Barcelona is proud to host the return meeting and the local organizers look forward to many participants and high-level talks. As said last year, our nations are closer to each other than one would expect. Thus we are following the steps towards a broader and more intense collaboration.

A novelty this year will be the Young Researchers Session on September 21. In this joint session, talks will be given by young researchers having participated either in the Czech-Slovak Research Competition for University Students or in the Évariste Galois Prize of the Catalan Mathematical Society.

We warmly thank all session organizers and members of the Organizing Committee for their efforts in this joint event, and look forward to very pleasant days during the conference.

Carles Casacuberta, SCM President
Jan Kratochvíl, CMS President

Acknowledgements

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Institut d'Estudis Catalans
Departament d'Universitats, Recerca i Societat de la Informació (Catalan Government)
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Programme Overview

Thursday, September 21

15.30 - 20.00 Young researchers session

Friday, September 22

9.00 - 10.15 Registration
10.15 - 10.30 Opening speeches
10.30 - 11.30 Plenary talk: Daniela Jarušková
Coffee break
12.15 - 13.15 Plenary talk: Paul Eklof
Lunch
15.30 - 17.00 Parallel sessions
Coffee break
17.30 - 18.30 Parallel sessions
20.00 Conference dinner
(Restaurant *El Cangrejo Loco*, at Port Olímpic; see the separate information.)

Saturday, September 23

9.30 - 11.00 Parallel sessions
Coffee break
11.30 - 12.30 Plenary talk: Demeter Krupka
12.30 - 13.30 Plenary talk:Josep Maria Font
Lunch
15.30 - 17.00 Parallel sessions
Coffee break
17.30 - 18.30 Plenary talk: Jiří Matoušek
18.30 - 18.45 Closing speeches

Rooms

Ground floor

Computational Statistics and Data Analysis	Sala Pere Coromines
Logic	Sala Pi i Sunyer
Mathematical Physics	Sala Nicolau d'Olwer

First floor

Plenary Talks	Sala Prat de la Riba
Young Researchers Session	Sala Prat de la Riba
Discrete Mathematics and Computer Science	Sala Prat de la Riba
Mathematical Analysis	Sala Massó i Torrents
Ring and Module Theory	Sala Puig i Cadafalch

Index of Abstracts

PLENARY TALKS

Set-theoretic methods in module theory

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We survey, for the non-expert, some uses of set-theoretic methods to prove theorems in module theory—largely in ordinary, Zermelo-Frankel set theory, without additional axioms. Applications discussed include Baer modules, tilting theory, and complete cotorsion pairs.

Compatibility in algebra, logic and computer science

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In this talk I will describe some recent developments in abstract algebraic logic by focusing on the role played by the notion of the compatibility of a congruence relation with respect to a unary relation (a subset). This notion has given rise to those of the Leibniz congruence and the Leibniz operator, which have been two of the key elements in these developments. It has also been applied in the analysis of the behavioural equivalence of hidden data structures, in the study of algebraic specifications of object oriented programming.

Change point methods for detection changes in temperature series

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During last twenty years statisticians developed many new procedures to detect non-stationarities and trends in time series. These new methods, that are widely known as "change point methods", are often applied to detect changes in annual mean temperature series.

In the scope of mathematical statistics the decision whether a series $\{Y_i\}$, observed sequentially at time points $i = 1, \ldots, n$, is stationary, is usually based on hypotheses testing. The null hypothesis claims that the series is stationary while the alternative hypothesis claims that there is a change(s). Moreover, the alternative corresponds to the type of change we are looking for. For instance, if we are looking for a change in location, the simplest change has a form of a sudden shift in the mean of $\{Y_i\}$ that occurs at some unknown time point. The other often considered type of change is a so-called gradual change. The simplest example of such a change

has a form of a linear trend in mean that appears at an unknown time point. In these two cases the maximum type test statistics, which may be derived from the likelihood principle, may be applied. To find critical values we need to know the distribution of the suggested test statistics under the null hypothesis. As their exact distribution is very complex and the number of observations is usually large, an asymptotic approach may be applied. This leads to the theory of extremes of Gaussian processes. It is interesting to notice that in the two upper situations we obtain two completely different processes with a different extreme behavior.

Changes in the mean may be accompanied by changes in some other characteristics of the series. For instance, many climatologists expect that extremal weather pattern, e.g. appearance of long hot spells, will become more common. This rises an interest in studying changes in the behavior of annual maximal and minimal temperatures or in numbers of exceedances over a given high threshold. Here again the asymptotic approach may be applied.

The suggested procedures will be applied to very long temperature series that were measured at several european observatories. The daily series that are mostly longer than two hundred years were taken from the book of Camuffo and Jones (2002).

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The inverse problem of the calculus of variations: local and global aspects

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This lecture is devoted to recent developments and some open problems of higher order global variational theory in fibred spaces. In the first part we discuss the classical Helmholtz conditions for a system of differential equations ensuring that these equations be the Euler-Lagrange equations of a variational functional (the inverse problem of the calculus of variations). We also discuss the Douglas' and Sonin's versions of these conditions. Then we give a general formula for systems of partial differential equations to be variational. In the second part of the talk we present basic concepts of variational analysis on topologically non-trivial fibred spaces. In terms of the variational sequence theory built from sheaves of differential forms, we give the cohomology conditions describing differences between local and global variationality of partial differential equations on manifolds. As examples we discuss the energy-momentum tensors and consequences of topological properties of underlying spaces for global variationality of the motion equations in higher order mechanics.

Zone diagrams

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A zone diagram is a new variation of the classical notion of Voronoi diagram. Given points (sites) $p_1, \ldots, p_n$ in the plane, each p_i is assigned a region R_i, but in contrast to the ordinary Voronoi diagrams, the union of the R_i has a nonempty complement, the neutral zone. The defining property is that each R_i consists of all points that lie closer (non-strictly) to p_i than to the union of all the other R_j. Thus, the zone diagram is defined implicitly, by a fixed-point property, and neither its existence nor its uniqueness seem obvious. We prove both, as well as convergence of a natural iterative algorithm for computing it. Many challenging questions remain open.

Joint work with Tetsuo Asano and Takeshi Tokuyama.

COMPUTATIONAL STATISTICS AND DATA ANALYSIS

Optimal Filtering in a Non-Linear Non-Gaussian Framework: Benchmark Study

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Filtering is an important task within many scientific areas. That is, many times one is interested in extracting knowledge about an unknown underlying state variable based only on data and some prior information. To solve this problem, many approaches have been developed. In this work, we study and apply the simulation based methodology called Particle Filtering, using sampling importance resampling (SIR), to estimate the underlying state in a non-linear and non-Gaussian framework. A Particle Filter (PF) is a sequential Monte Carlo method which allows to produce samples from a target distribution as new observations arrive.

We present a benchmark study in order to assess the filtering performance of various PF variants. To do so, we use a synthetic non-linear non-Gaussian dynamic model already considered in the literature and we apply four PF variants to recursively estimate the states. These PF variants are: Kitagawa's SIR PF, Pitt and Shephard's auxiliary sampling importance resampling (ASIR) PF, and the so called extended PF (EPF) and unscented PF (UPF). The EPF and UPF emerge from combining, respectively, the extended Kalman Filter (EKF) and the unscented Kalman Filter UKF with the general SIR PF. The non-simulation based filters, EKF and UKF, are also included in the benchmark study. Additionally, the deterministic and residual resampling schemes are compared and the effect of the number of particles is checked. All the mentioned filters have been implemented by us using the R-language. Following the simulation study, results and conclusions are presented considering both statistical and computational efficiency.

Finally, some words regarding the simultaneous estimation of state and parameters are stated. For instance, Liu and West (2001) use a modified ASIR strategy to get a combined estimation of state and parameters. In 2003, we proposed a modified SIR PF combining ideas from Kitagawa and Liu and West. A further aim of this study is thus to extrapolate ideas of EPF and UPF to perform simultaneous estimation of state and fixed parameters.

Statistical matching methodologies

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Statistical Matching, also known as Data Fusion, involves the imputation of a complete block of missing variables in independent files. The interest of statistical matching is increasing every

day. It has been applied for a long time in entrepreneurial contexts to merge information coming from independent sources. Its main application is in media studies, where they are used to combine consumption panels with audience panels, but also in official statistics where they are used to integrate the information coming from different surveys or between a survey and a registry, entailing a reduction of cost.

We address this problem in its simplest case, which there are two files, one with complete information (X, Y), called donor file, and the other with incomplete information (X), called the recipient file, being the objective to fill in (to impute) the missing information Y in the latter file. Although we assume a MAR pattern among the specific variables respect to the common ones, we can't in general assume that both files are representative samples of the same population with iid observations. Whatever be the case, the main goal of statistical matching is to reproduce the distribution of the complete information in the recipient file.

The quality of the fused data can be assessed by means a series of statistics referring the preservation of the marginal statistics and the joint distribution of variables, but even though these objectives are fulfilled, they don't allow treating the complete fused file as a complete case data, since it doesn't take into account of the imputation uncertainty.

There are three basic approaches for statistical matching. The first one consists of embedding the common and specific variables within a *parametric* multivariate distribution $f(X, Y/\theta)$, assuming donors and receptors independent random draws from this distribution. This distribution can be factored into $f(X, Y/\theta) = f(Y/X, \theta_{Y/X})f(X, \theta_X)$; hence, it is possible to estimate its parameters θ_X and $\theta_{Y/X}$ from the available information and using them to impute the missing block of data; the second approach consists in directly *modelling* the relationship between the Y variables with the X variables in the donor file: $f(Y/X) = r(X) + \varepsilon$ (*explicit modelling*) and applying this model in the recipient file; and the last approach consists in finding for each individual of the recipient file one or more donor individuals as similar as possible, and then in some way, transferring the values of the Y variables to the *recipient* individual (*implicit modelling*). This method is known as *hot deck*, a term borrowed from data editing in data bases.

We will present a comparison of the aforementioned three main methodologies of imputation applied to an actual operation of combining Internet data with household survey data. Household surveys are carried out by statistical offices following a careful sampling design. Internet data flows continuously through the net, it concerns the usage of the different resources, URL's, pages, etc. by the users. With special devices it is possible to collect this information, taking into account of the confidentiality laws. This latter source of information is of course not random, but it could be fruitful to fusion with the household survey information. Moreover, in the context of the application, we address the problem of the validation of the envisaged fusion.

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A note on estimation of reliability for non observed data

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Every components manufacturer must not only respect reliability clauses stated in the contract but also be in a position to match reliability requirements in the bids. These requirements essentially concern two closely connected reliability parameters, i.e., the failure rate and the mean time between failures. Those parameters are usually estimated by appealing to formulae from standards, most of which are nowadays obsolete. This usually leads to "over-pessimistic" estimation. For correcting these predicted values it is necessary to keep track of the operating equipments in order to evaluate their operational or observed reliability.

The main aim of this paper is to offer an analytic model permitting to access the operational reliability of highly reliable equipments in a non common context when the only available information is the number of equipments which were delivered during each period and the number of the equipments removed during each period.

This is a joint work with my colleagues J. Berthon, J.-M. Deshouillers and Y. Dutuit from the University Bordeaux 1.

Equivalence of the ROC curves

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Receiver operating characteristics (ROC) curves are a useful instrument for evaluation of the performance of supervised classifiers. Despite the fact that the simple form of ROC curves is applicable only to the two-state classification, it covers a broad spectrum of applications as clinical diagnostics in medicine, computational linguistics, machine learning, data mining, etc.

This contribution will concentrate on the concept of equivalence of the ROC curves and its usage to distinguish among classifiers. There will be presented several parametric and non-parametric approaches suitable for testing the difference between ROC curves. Results will be illustrated on data sets coming from the sociological research.

Fitting a volatility-varying and jump-difussion Merton type model to interest rate data from the Spanish market

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We present a generalization of the traditional Lévy-Merton jump diffusion model, allowing stochastic volatility. In order to estimate jump instants and jump amplitudes we use and improve

a method based on the quadratic variation. We apply this methodology to two time series provided by the "Banco de España" with daily observations of the interest rate for operations of 1 day and 1 year (from 4th January 1988 to 31st December 1998).

JEL Classification: C13, C22, C51, E49.
Keywords: Levy processes, Merton model, jump-diffusion models, interest rate.

Tree-based detection of microsleeps from EEG spectra

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Classification of EEG spectra focused at detection of somnolence is highly important for, e.g., prevention of traffic accidents, and, at the same time, complicated. Apart from primary data, i.e., spectral powers from different channels (locations) and frequency bands, many derived variables—so called secondary data—should be used. The latter are, for instance, sums of powers in adjacent spectral bands, and differences between, or ratios of some primary or secondary data. This makes the dimension of predictor space high, and the predictor variables heavily correlated. Classification trees and, namely, forests represent promising methods for such data-analytic tasks.

During the presentation, performance of various tree-based classification techniques will be compared, using EEG data from a driving simulator. Apart from other results, it will be demonstrated that a proper choice of the spectra type (i.e. of the way the "raw" EEG data are transformed into the spectra) is as important for the precision of the classifier as the methods of further spectra analysis.

Evolution of imitation networks in minority game model

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We investigate *minority game with imitation* placed on a complex network. The network is only substrate of the real interactions leader-follower what is set up by playing minority game. There were investigated 2 types of substrate networks—linear chain and Barabási-Albert network. Payment for imitation changes the structure of imitation networks in both cases and strongly benefit highly connected agents for the high payments.

Predictive Ability of Threshold Volatility Models

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The aim of this paper is to compare the forecasting performance of competing volatility models, in order to capture the asymmetric effect in the volatility. We focus on examining the relative out-of-sample forecasting ability of the models (SETAR-TGARCH and SETAR-THSV), which contain the introduction of regimes based on thresholds in the mean equation and volatility equation, compared to the GARCH model and SV model. For each model, we consider two cases: Gaussian and *t-Student* measurement noise distribution. An important problem when evaluating the predictive ability of volatility models is that the "true" underlying process is not observable and thus a proxy must be defined for the unobservable volatility. To attain our proposal, the proxy volatility measure and the loss function must also be decided to ensure a correct ranking of models.

We developed a specific code in the R language for the estimation of models; see the web page `http://cran.r-project.org`. For all models, a simulation study was carried out using as true parameters values those already published in the literature. This is done in order to assess the accuracy and stability of our implemented algorithms and thus guarantying the quality of our results.

In order to compare the forecasting performance of the models we have studied two return time series: IBEX 35 and S&P500. Our empirical application suggests the following results: We have obtained, taking into account the MSE statistics, a different best model for each case. For the IBEX 35, the introduction of threshold in the mean and variance equations produces more accurate predictions. The best model to forecasting the volatility of IBEX 35 is the SETAR-THSV model . In the S&P500 return series, however, the leverage in the mean is not important and thus the SV_t model is flexible enough to beat the threshold models.

Use of likelihood criteria for the verification of realistic simulation models

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Introduction. Neter et al. (1996) describe a clinical trial about the limitation that rheumatic arthritis produces in the mobility of the affected persons. In this trial, three treatments were administered during 14 weeks to 57 patients (20 placebo (p), 16 standard treatment (s) and 21 new treatment (t)). Using linear mixed effects models, the authors demonstrated a greater efficacy of the new treatment. The main variable was a scale of mobility (1-100%).

Aims. The aim of this study is to complete the modelling results in Neter et al. (1996). More precisely, we build a representative computational model, reproduce the trial by means of Montecarlo simulation and, using these simulations illustrate some statistical methods for the verification of the computational model (Monleón, 2005). These methods are based on a goodness of fit (GOF) criterion of the simulation replicates with respect to the conceptual

estimated model. In this context, we opted for the use of a -Log (Likelihood) GOF criterion (-LL), more optimistic when there are many model parameters, if compared to other criterions like AIC (Akaike's Information Criterion) or BIC (Schwartz's Bayesian criterion).

Results. 100 clinical trial simulation replicates were generated using SAS language and analysed by means of this methodology, based on the percent variation of the above cited likelihood criteria along the simulations replications, compared to the conceptual model. The percentage of -LL variation for the 100 simulation replicates, with regard to the likelihood of the conceptual model were calculated. It was observed that the percentage of variation for -LL (PVLL) was in the range from 0.0265-6.6730%. This is a commonly accepted range for simulation replicates of a computational model truly verifying the conceptual model. Our own experience with referenced clinical trials suggests a 0 to 20% range (Monleón, 2005) can be good to a computational model that verifies the conceptual model. Conclusions. PVLL seems to be a valid indicator in order to verify the simulation model versus the conceptual model, in a suitable form. One recommends to use this test and the range of oscillation for the PVLL in all simulations for verification of the computational model in front conceptual model as a first test and secondly use other, more elaborated, statistical techniques like to confront the confidence interval obtained in the simulations and those of the conceptual model.

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Regression in Sobolev spaces

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We propose a class of nonparametric estimators for the regression models based on least squares over the sets of sufficiently smooth functions. Least squares permit the imposition of additional constraint–isotonia–on nonparametric regression estimation and testing of this constraint.

The estimation takes place over the balls of functions which are elements of a suitable Sobolev space–special types of Hilbert spaces that facilitate calculation of the least squares projection. The Hilbertness is allowing us to take projections and hence to decompose spaces into mutually orthogonal complements. We change the proof of very important *Representors in sobolev space theorem* from [1]. We are examining representors' properties and prove a theorem, which provides the way of construction of the representors and their exact form.

Then we transform the problem of searching for the best fitting function in an infinite dimensional space into a finite dimensional optimization problem–quadratic optimization with quadratic and linear constraints. We generalize this theorem into the weighted regression case. Then we investigate the form of the regression estimator in the Sobolev space. Hence we also

prove symmetry and positive definiteness of the representor matrix. We use Schur Decomposition Theorem to solve our optimizing problem and prove the existence and the uniqueness of its solution. We prove the existence of 1–1 mapping between the Sobolev bound and the smoothing parameter.

We prove that the balls of functions in Sobolev space are bounded and have bounded higher order derivatives. It permits us to estimate over rich set of functions with sufficiently low metric entropy and apply laws of large numbers and central limit theorems. We also declare two main attitudes to isotonia–definite and indefinite. Finally, we implement asymptotic behavior of the regression estimator and bootstrap techniques into the algorithms.

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Resampling Generalized Linear Mixed Models

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Generalized Linear Mixed Models allow to deal with non-normal response, like binary or count data. These models take into account dependent structure of grouped data coming from repeated measurements or longitudinal data. Although this methodology has a wide range of application, estimation procedures usually rely on numerical approximations (Laplace, PQL). Therefore, inference from classical approaches is sometimes inaccurate or has lack of power.

An R library which implements generation of bootstrap data and efficient fit of GLMM via PQL is presented. Several options can be used to generate the resampled data: parametric or empirical resampling of random effects and generation according to the theoretical distribution from the linear predictor, reconstruction of data from different residual resampling with the corresponding truncation, with or without the nested structure between random effects and residuals. Fitting the resampled data, confidence intervals and empirical p-values based on the bootstrap methodology can be obtained, for fixed effects and for variance components.

With the same library, a simulation study has been developed in order to compare the performance of different configurations of bootstrap procedures in presence of misspecification of distribution of random effects and/or residuals for logistic and Poisson regression.

On confidence intervals for binomial parameter in case of zero observed frequency

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In biomedical applications of statistics, the "standard" confidence interval (CI) based on normal approximation and Wald statistic prevails as an event probability estimation technique. When the observed event frequency is low, and namely in case of no event occurrence (a situation often

met in medical studies), the "standard" CI is usually substituted with the "exact" (Clopper-Pearson) CI.

The "standard" CI is known to have a poor coverage probability $C(p)$ near the extremes 0 and 1 ([2]), and it degenerates into a single point when the observed event frequency k is zero (or n out of n). The "exact" CI, on the other hand, is over-conservative under analogous circumstances, with $C(p) \geq 1 - \alpha/2$ near the limits (the nominal coverage probability being $1-\alpha$). This suggests the idea of modifying the "exact" CI, and applying the nominal confidence level of $1 - 2\alpha$ for $k = 0$.

The above modification of the "exact" CI is not correct in sense that, unlike the unmodified CI, it does not guarantee that $C(p) \geq 1 - \alpha$ for all p (see, e.g., [3]).

We shall, nevertheless, reconsider the idea in the context of a recent work [1] by Agresti and Gottard. They found a new attractive motivation for so called *mid-P confidence interval*, which yields for $k = 0$ the same confidence limits as the modified Clopper-Pearson CI.

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DISCRETE MATHEMATICS AND COMPUTER SCIENCE

On the order of bi-regular cages of even girth

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By a bi-regular cage we mean a graph on a minimum number of vertices with degree set $\{r, m\}$ and girth g. In this work we improve the known lower bound for the order of a $(\{r, m\}; g)$-cage denoted by $f(\{r, m\}; g\})$ for even girth $g \geq 6$ proving the following theorem.

Theorem 1. The order of a $(D; g)$-cage with $D = \{r, m\}$ where $3 \leq r < m$ and even girth $g \geq 6$ is

$$f(\{r,m\};g\}) \geq \begin{cases} m+2+(mr-2)\dfrac{(r-1)^{g/2-2}-1}{r-2}+(r-2)(r-1)^{g/2-2}, \text{ for } r \geq 4; \\ 1+\dfrac{(7m+3)2^{g/2-2}}{3}-m, \text{ for } r=3. \end{cases}$$

Moreover we obtain the following result which supports the conjecture that $f(\{r, m\}; 6) = 2(rm - m + 1)$ for any $r < m$, posed by Yuansheng and Liang (*The minimum number of vertices with girth 6 and degree set $D = \{r, m\}$*, Discrete Mathematics 269 (2003), 249–258).

Theorem 2. Let $r, k \geq 2$ be two integers such that $r - 1$ is a prime power. Then

$$f(\{r, k(r-1)+1\}; 6) = 2k(r-1)^2 + 2r.$$

Joint work with G. Araujo, P. García-Vázquez, X. Marcote and J. C. Valenzuela.

Linear combinations of permutation polynomials

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Let p be a prime. Let $\mathbb{F}_p$ be the finite field with p elements. We say $f \in \mathbb{F}_p[X]$ is a *permutation polynomial* if the map $x \mapsto f(x)$ is a permutation of $\mathbb{F}_p$.

In 1970 Rédei and Megyesi proved that if there are more than $(p-1)/2$ elements a with the property that $f(x) + ax$ is a permutation polynomial then $f(x) + cx + d = 0$ for some $c, d \in \mathbb{F}_p$. In other words, the graph of f,

$$\{(x, f(x)) \mid x \in \mathbb{F}_p\},$$

is (contained in) a line.

In 2003 Gacs proved that if there are more than $2\lceil\frac{p-1}{6}\rceil + 1$ elements a with the property that $f(x) + ax$ is a permutation polynomial then $(f(x) + cx + d)(f(x) + bx + e) = 0$ for some $b, c, d, e \in \mathbb{F}_p$. In other words, the graph of f is contained in the union of two lines.

In joint work with Gacs and Sziklai we have shown that if there are more than about $2p^2/9$ pairs $(a, b) \in \mathbb{F}_p^2$ with the property that $f(x) + ag(x) + bx$ is a permutation polynomial, then there are elements $c, d, e \in \mathbb{F}_p$ such that $f(x) + cg(x) + dx + e = 0$.

I will give an indication of the techniques used in the proof and give an example of polynomials f and g for which there are about $2p^2/9$ pairs $(a, b) \in \mathbb{F}_p^2$ with the property that $f(x)+ag(x)+bx$ is a permutation polynomial but there are no elements $c, d, e \in \mathbb{F}_p$ such that $f(x)+cg(x)+dx+e = 0$. I will also make a couple of conjectures and briefly consider the corresponding problem for polynomials in many variables.

Block transitivity and degree matrices

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We say that a square matrix M is a degree matrix of a given graph G if there is a so called equitable partition of its vertices into r blocks such that whenever two vertices belong to the same block, they have the same number of neighbors inside any block.

We ask now whether for a given degree matrix M, there exists a graph G such that M is a degree matrix of G, and in addition, for any two edges e, f connecting the same pair of blocks there exists an automorphism of G that sends e to f. In this work, we fully characterize the matrices for which such a graph exists and show a way to construct one.

On Two Zero-Sum Conjectures of Gao, Thangadurai and Zhuang

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Gao, Thangadurai and Zhuang conjectured that any sequence of terms from the cyclic group of order m of length at least $|S| = m + m/p - 1$, where p is the smallest prime divisor of m, must either contain an m-term subsequence whose terms sum to zero, or else a term whose multiplicity is at least $|S| - m + 1$. By confirming and generalizing a related conjecture of Gao et al., we are able to confirm this result, and, moreover, we extend the result to an arbitrary abelian group.

Joint work with Ordaz, Varela and Villarroel.

Closure concepts in claw-free graphs

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In 1970's, Bondy and Chvatal introduced a closure concept of graphs based on Ore's lemma. In 1997, Ryjacek introduced a closure concept in claw-free graphs based on a local completion at a locally connected vertex. The stability of several properties of graphs were proved. We present a variation of this closure concept, called the edge-closure, which is based on a local completion at a locally connected edge. Similarly to Ryjacek, we prove stability of the circumference of graphs, and we show stability of some classes of graphs defined in terms of forbidden induced subgraphs.

On the Number of Plane Geometric Graphs

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We investigate the number of plane geometric, i.e., straight-line, graphs, a set S of n points in the plane admits. We show that the number of plane geometric graphs and connected plane geometric graphs as well as the number of cycle-free plane geometric graphs is minimized when S is in convex position. Moreover, these results hold for all these graphs with an arbitrary but fixed number of edges. Consequently, we provide a unified proof that the cardinality of any family of acyclic graphs (for example spanning trees, forests, perfect matchings, spanning paths, and more) is minimized for point sets in convex position.

In addition we construct a new extremal configuration, the so-called double zig-zag chain. Most noteworthy this example bears $\Theta^*\sqrt{72}^n = \Theta^* 8.4853^n$ triangulations and $\Theta^* 41.1889^n$ plane graphs (omitting polynomial factors in both cases), improving the previously known best maximizing examples.

Joint work with O. Aichholzer, T. Hackl, C. Huemer, H. Krasser and B. Vogtenhuber.

Recent results and open problems in intersection graphs: A short survey

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Geometric intersection graphs are studied both for their practical motivations and for interesting theoretical properties. We will survey recent results concerning recognition of certain classes of intersection graphs (namely so called string graphs and polygon-circle graphs), and describe the development in some other long standing open problems in the area (representability of planar and co-planar graphs, complexity of some optimization problems etc.).

Problems on games: complexity versus succintness

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We are interested in the study of the computational complexity of problems involving games with a large number of player's or actions. In particular in deciding the existence of a Pure Nash Equilibrium or deciding whether two strategic games are isomorphic. For doing so it is fundamental to decide how to represent a game as the input to a computer. We propose three ways of representing a game, each of them with different degree of succinctness for the components of the game and show how the computational complexity of the problems changes depending on the representation.

Joint work with Carme Àlvarez, Joaquim Gabarró, and Alina García

Ramsey Theory and Combinatorial Games

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Ramsey theory studies the internal homogeneity of mathematical structures (graphs, number sets), parts of which (subgraphs, number subsets) are arbitrarily coloured. Often, the sufficient object size implies the existence of a monochromatic sub-object. Combinatorial games are 2-player games of skill with perfect information. The theory of combinatorial games studies mostly the questions of existence of winning or drawing strategies. Let us consider an object that is studied by a particular Ramsey-type theorem. Assume two players alternately colour parts of this object by two colours and their goal is to create certain monochromatic sub-object. Then this is a combinatorial game. We focus on the minimum object size such that the appropriate Ramsey-type theorem holds, called *Ramsey number*, and on the minimum object size such that the first player has a winning strategy in the corresponding combinatorial game, called *game number*. In this talk, we study the *restricted Ramsey theorems,* where the Ramsey number is substantially greater than the game number. This means, we show the existence of first player's winning strategies, together with the game number upper bound, and we compare it with the appropriate Ramsey number. The first part of Whitehead algorithm made polynomial Enric Ventura.

The classical Whitehead algorithm to check whether two words in a free group belong to the same orbit, has two separated steps which are both typically polynomial on the length of the words, and exponential on the ambient rank. However, some heuristics, partial results and computational experiments indicate that, in practice, the algorithm it is faster than this. We present a modification of this classical algorithm which is polynomial on both the length and the rank ambient. The new algorithm is surprisingly simple, making use of a classical argument about Whitehead automorphisms and Whitehead graphs, and the classical Max-Flow Min-Cut algorithm for graphs. As an application, we obtain a polynomial algorithm to check wether two given subgroups of a free group are one a free factor of the other.

Joint work with P. Weil and A. Roig.

Paths with no small angles

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Answering a question of Dumitrescu we show that given a finite set of points in the plane, it is possible to connect them to a (possibly self-intersecting) polygonal path so that every angle on the polygonal path is at least $\pi/9$.

Joint work with I. Barany and A. Por.

LOGIC

Indistinguishability-based graded properties of fuzzy relations

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The traditional definitions of reflexivity, symmetry, etc., of fuzzy relations are given by a crisp condition (e.g., reflexivity by $Rxx = 1$ for all x). *Graded* definitions are given by a formula of a suitable fuzzy logic—e.g., $(\forall x)Rxx$ for reflexivity (in, say, MTL$\triangle$ [4]). The defining formulae are essentially the same as the classical ones, fuzziness being introduced by the semantics of connectives and quantifiers in fuzzy logic. Graded properties of fuzzy relations have been introduced in Gottwald's paper [5] and studied by several authors since [6, 7, 3, 8].

A full gradedness of the properties of fuzzy relations requires substituting a fuzzy similarity (indistinguishability) E, not only for the explicit occurrences of crisp identity $=$ in the defining formulae (e.g., of antisymmetry or functionality), but also for multiple references to the same variable (which in fact implicitly require the crisp identity of the denoted objects). This leads to E-based definitions of these properties, e.g., E-reflexivity $\forall(xx')(Exx' \to Rxx')$, E-symmetry $\forall(xx'yy')(Exx'\&Eyy'\&Rxy \to Ry'x')$, etc. These have been introduced by the present author in [1], where it has been shown that under the non-graded approach (when required to degree 1), E-properties reduce to the traditional ones plus the extensionality of R w.r.t. E; however, when graded, they differ and have to be investigated separately. In the present talk, further details will be added to [1], focusing especially on the properties of E-functions, the variants of E-properties depending on the requirements on indistinguishable witnesses of the properties, and the generalization of this approach to non-contractive substructural logics in general (as the prelinearity condition that characterizes fuzzy logics [2] is not important in these considerations).

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Uniform interpolation in modal provability logics

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Since Craig's landmark result on interpolation for classical predicate logic, proved as the main technical lemma in [1], interpolation is considered one of the central concepts in pure logic. Various interpolation properties find their applications in computer science and have many deep purely logical consequences. We focus on a stronger propositional version of Craig interpolation property – the uniform interpolation property – which arises in relation with quantifier elimination:

> For every formula A and any choice of propositional variables $\bar{q}$, there is a *post-interpolant* $I_{postA}(\bar{q})$ such that for all B, whenever $A \to B$ is provable and the shared variables of A and B are among $\bar{q}$, $A \to I_{postA}(\bar{q})$ and $I_{postA}(\bar{q}) \to B$ are provable. Symmetrically, there is a *pre-interpolant*.

As usual in the case of modal logics, semantic methods are preferred proving important characterizations of a logic. Semantic proofs of uniform interpolation for modal propositional logics based on a simulation of propositional quantifiers were given by Visser in [3] for modal logics **K**, Gödel-Löb's logic of provability **GL** and Grzegorczyk's logic **S4Grz**. For **GL**, uniform interpolation was first proved by Shavrukov in [5].

Our approach is proof-theoretic: from this point of view Craig interpolation relates to cut-free proofs, while uniform interpolation relates to terminating proof-search trees. We use a method introduced by Pitts in [4]. The argument uses a simulation of propositional quantifiers in the framework of an analytic terminating sequent calculus, i.e., a proof system in which any backward proof-search terminates. The main motivation for applying the proof-theoretic method is that it provides an explicit effective and also easily implementable construction of uniform interpolants. So far such a proof-theoretic proof has been given by the author for modal logics **K** and **T** in [2].

We prove uniform interpolation theorem for two modal logics having an arithmetical interpretation - Gödel-Löb's logic of provability **GL** and Grzegorczyk's logic **S4Grz**. The former yields an alternative constructive proof of the fixed point theorem for **GL**. The main point of the proof is in the use of a terminating proof system. While in the case of **GL**, its standard sequent calculus is sufficient in this sense, in the case of logic **S4Grz** we are forced to use a loop-preventing mechanism built into the syntax to enforce the termination of our procedure.

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An embedding removing propositions and preserving validity for modal logic $\boldsymbol{K}$

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We will show a polynomial time reduction from the (minimal) modal logic $\boldsymbol{K}$ with a countable number of propositions to $\boldsymbol{K}$ without any proposition. Indeed, we will prove that the map $\mathsf{t} : \mathcal{L}^{mod} \longrightarrow \mathcal{L}^{mod}$ defined by the following clauses:

$$\begin{array}{lcl} \mathsf{t}(\bot) & := & \bot \\ \mathsf{t}(\top) & := & \top \\ \mathsf{t}(p_n) & := & \Diamond(\Box\Diamond\top \wedge \Diamond^{n+2}\Box\bot) \\ \mathsf{t}(\sim\varphi) & := & \sim\mathsf{t}(\varphi) \\ \mathsf{t}(\varphi_0 \wedge \varphi_1) & := & \mathsf{t}(\varphi_0) \wedge \mathsf{t}(\varphi_1) \\ \mathsf{t}(\Box\varphi) & := & \Box(\Diamond\Box\bot \supset \mathsf{t}(\varphi)) \end{array}$$

satisfies that φ is in $\boldsymbol{K}$ iff $\mathsf{t}(\varphi)$ is in $\boldsymbol{K}$. From this fact we will be able to obtain new proofs for determining the complexity of certain modal logics without propositions.

Fuzzy logics, proof theory, and normal forms

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A method is described for obtaining conjunctive normal forms using Gentzen-style rules possessing a special kind of invertibility. This method is applied to several important fuzzy logics using hypersequent rules adapted from those defined in the literature (see [1, 2, 3, 9]). In particular a conjunctive normal form with simple McNaughton functions as literals is generated for Łukasiewicz logic (see [11]), and normal forms with simple implicational formulas as literals obtained for Gödel logic, Product logic, and Cancellative hoop logic (see [3, 4, 7]).

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Generalization of Mundici's functor to linearly ordered involutive monoidal t-norm based logics IMTL

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In this note we present a generalization of Mundicis functor Γ between MValgebras and ordered abelian groups with strong unit (see [4]) to linearly ordered IMTL-algebras. The generalization shows that the associativity in the algebraic ordered structures associated to IMTL-chains is lost (and cancellativity as well). Moreover the strong unit used in Mundicis functor is required to have stronger properties. In the talk we define what we call partially associative ordered grupoid with strong associative unit (PAOG) and prove that are the algebraic counterpart of linearly ordered IMTL in the following sense: We define two mappings, $f : IMTL \rightarrow PAOG$ and $g : PAOG \rightarrow IMTL$ and we prove that, Given a IMTL algebra A, then $A \equiv g(f(A))$ and Given a PAOG G, then $G \equiv f(g(G))$. Therefore the algebraic structure (the PAOG) associated to each IMTL-chain seems to be "adequate" to our purposes. The generalization to the non-linear case seems not difficult but in any case will be a good test for knowing the adequacy of the defined algebraic ordered structure.

Applications of totally ordered monoids which are not formally integral in many-valued logic

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A totally ordered commutative monoid is said to be formally integral if it is a quotient of some totally ordered free commutative monoid. This notion was introduce in [2] where the authors gave several criteria how to recognize formally integral monoids and presented also examples of monoids which are not formally integral. In this talk we are going to show that these results can be useful in many-valued logic. We will present three applications of an example from [2]. In the first application we use the example for proving that the product Łukasiewicz logic does not enjoy the standard completeness theorem. The second application coming from [4] uses the result showing that not all weakly cancellative MTL-chains can be viewed as trunctions of ΠMTL-chains. Finally, we will discuss the third application from [1] where we prove that the implicational fragment of the logic ΠMTL is not the same as the implicational fragment of the monoidal t-norm based logic (MTL).

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Analysis of the implication and negation-less fragments of the logic of residuated lattices and some of its consequences for fuzzy logic

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We prove that the $\langle \vee, \wedge, *, 0, 1\rangle$-fragment of the the logic of residuated lattices (also known as intuitionistic logic without contraction, or monoidal logic) is exactly the $\langle \vee, \wedge, *, 0, 1\rangle$-fragment of classical logic. Analogous results are proved concerning the $\langle \vee, \wedge, 0, 1\rangle$-fragment and the $\langle \vee, *, 0, 1\rangle$-fragment of the the logic of residuated lattices. These results generalize the well known result stating that the $\langle \vee, \wedge, 0, 1\rangle$-fragment of intuitionistic logic is exactly the $\langle \vee, \wedge, 0, 1\rangle$-fragment of classical logic.

As a consequence, the $\langle \vee, \wedge, *, 0, 1\rangle$-fragment (and also the other two fragments) of every t-norm based fuzzy logic is exactly the $\langle \vee, \wedge, *, 0, 1\rangle$-fragment of classical logic. This result

generalizes the well known result concerning the corresponding fragments of Gödel logic and classical logic.

The talk will include some comments about truth preserving t-norm based fuzzy logics and t-norm based fuzzy logics preserving degrees of truth.

Keywords: Substructural logics, t-norm based logics, residuated lattices, semilatticed monoids, Gentzen systems.

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A logic based on the ordinal sums of Łukasiewicz t-norms

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BL (Basic Logic) is the logic based on all continuous t-norms and their respective residua. As it is well-known, we may equally consider BL as the logic of all finite ordinal sums of Łukasiewicz t-norms and their residua only (see e.g. [Mon]). We consider here the logic mŁL to which this latter explanation applies as well, but in a narrower sense. Namely, the language of BL is enriched by an unary connective whose interpretation is *the largest idempotent below*.

Whereas the proof theory for BL contains specific difficulties, the situation seems to be better for mŁL. A proof system for mŁL may be formulated on the base of hypersequents in a way that the rules involving the connectives are invertible. Our work grew out of the work on hypersequent formulations of the standard extensions of BL; see in particular [CiFeMe].

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MATHEMATICAL ANALYSIS

Lyapunov exponents and the dimension of the attractor for 2D flow

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In the talk we explain how to estimate dimension of the attractor by the method of Lyapunov exponents. We will illustrate it on 2D Navier Stokes system describing flow of Newtonian fluid. Next we show where appear the main problems if we consider the generalized Newtonian fluid, how to overcome them and we present some recent results.

Optimal Sobolev embeddings

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Transference methods applied to multiplier theory

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In this talk, we present some results on the problem of restriction of Fourier multipliers. The techniques used are the so called *Transference methods* introduced by R. R. Coifman and G. Weiss.

MATHEMATICAL PHYSICS

Variational principles for locally variational forms

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We present the theory of higher order local variational principles in fibered manifolds, in which the fundamental global concept is a locally variational dynamical form. Any two Lepage forms, defining a local variational principle for this form, differ on intersection of their domains, by a variationally trivial form. In this sense, but in a different geometric setting, the local variational principles satisfy analogous properties as the variational functionals of the Chern-Simons type. The resulting theory extends the first order theories to higher order cases. This is a joint work with D. Krupka.

On homogeneous geodesics on homogeneous pseudo-Riemannian manifolds

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Homogeneous geodesics on Riemannian homogeneous manifolds were studied by many authors and many results were obtained. For pseudo-Riemannian manifolds, light-like homogeneous geodesics have a new interesting feature. We present the characterisation of homogeneous geodesics on pseudo-Riemannian manifolds, we show the interesting behaviour of light-like homogeneous geodesics on the example and we present examples of pseudo-Riemannian g.o. manifolds (where all geodesics are homogeneous) and pseudo-Riemannian almost g.o. manifolds (where almost all geodesics are homogeneous). This is a joint work with O. Kowalski.

Quantum waveguides: localized modes in twisted tubes

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In this talk we discusses Dirichlet Laplacian in a screw-shaped region, i.e. a straight twisted tube of a non-circular cross section. It was shown recently that, in contrast to a bending, the twisting acts as a sort of repulsive interaction. We demonstrate that a local perturbation which consists of "slowing down" the twisting in the mean gives rise to a non-empty discrete spectrum.

Non-linear realizations and branes

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We introduce the theory of non-linear realizations to describe the actions of bosonic and supersymmetric p-branes. We will also analyze how the diffeomorphism and kappa transformations are naturally incorporated in this framework.

A geometric approach to the Hamilton–Jacobi equation

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A geometric study of the Hamilton–Jacobi equation is performed both in the phase and also in the velocity space of a lagrangian system. In this general framework the problem is to describe the dynamics as integral curves of a family of first-order vector fields on the configuration space. One of the goals of this formulation is to deal with alternative lagrangian descriptions of a dynamical system.

Geometrical theory of non-holonomic systems applied: two rolling cylinders

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The geometrical theory of non-holonomic constrained system developed by Krupková [1] is applied to a practical situation: The non-holonomic mechanical system is made up by two long cylinders â a solid one put in a hollow one. The internal cylinder rolls on interior face of the hollow one. The external cylinder rolls on an underlay that forms with horizontal plane time depending angle. The non-holonomic constraint defined by the condition that both cylinders roll without slipping is derived. Theoretical analysis based on the theory of Krupková is made and the computer simulation (solution of reduced equations) is performed for this system. Physical interpretation of results is presented.

Constrained systems in nonholonomic mechanics and field theory

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Non-holonomic constraints in mechanics can be modelled as fibred submanifolds of the first jet bundle of a fibred manifold, naturally endowed with the so called canonical distribution.

This approach enables one to represent constrained systems by means of exterior differential systems defined on the constraint submanifold. We present constrained motion equations and discuss several important questions, such as structure of solutions, existence of a Lagrangian, conservation laws, regularity, and Hamilton form of the constrained equations. We also touch some recent ideas and results on generalizations to field theory.

An introduction to Quantum Information

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Quantum Mechanics is experiencing a renewed life. A plethora of new and surprising phenomena have been (re)discovered and implemented experimentally. The peculiarities of Quantum Mechanics, a cause of decades of philosophical debate, are nowadays used efficiently in communication and computation processes and have boosted the appearance of a new field known as Quantum Information. In this talk I will present some very basic elements of Quantum Mechanics which are at the heart of the new applications: entanglement, parallelism, etc. I will then discuss some interesting applications: cryptography, teleportation and dense coding. In the second part of the talk a will discuss some formal aspects of Quantum Information that may be of interest to mathematicians.

Generalized derivations and invariant functions of Lie algebras

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We work in the overall framework of so called generalized derivations of Lie algebras. We fully explore one of its special cases by constructing parametric subspaces of operators and operator Jordan and Lie algebras. Complete discussion of such a special case allows us to construct interesting complex invariant functions. We demonstrate usefulness of these functions on examples. These results are a contribution to isomorphism problem and are especially useful when investigating the parametric continua of Lie algebras. This is joint work with J. Hrivnák.

Minimum uncertainty states and optimal states satisfying a constraint

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In the quantum-mechanical measurement of the z component of the angular momentum, which is to inform of a rotation about the y axis, optimal are, in a well defined sense, the minimum-uncertainty states with respect to the incompatible y and z components of the angular momentum. An optimum of the Fisher information on a small rotation about the y axis is achieved

for similarly defined states, while the restriction to states with a given mean z component is essential to the definition. This is a joint work with A. Lukš.

The generating conjecture and Kinnersley's transformation

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By means of Ernst complex potential formalism it is shown, that some studied static axisymmetric Einstein-Maxwell fields obtained though the application of the Horský-Mitskievitch generating conjecture represent a combination of Kinnersley's transformations [W. Kinnersley: J. Math. Phys. **14** (1973) 651]. New theoretical background for the conjecture is suggested and commented. This is a joint work with Jan Horský and Jan Novotný (Masaryk University, Brno).

Hamilton equations related with second-order Euler–Lagrange forms

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The aim of the lecture is to announce some recent results concerning Hamiltonian field theory on fibred manifolds. The theory is based upon Lepagean equivalent of Euler–Lagrange form (Krupková 2001). The concepts of Hamiltonian system, Hamilton equations, regularity are presented. The case of second order Euler–Lagrange form non-affine in the second derivatives is studied.

Nonholonomic constraints and principle of virtual work

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The classical physical formulation of the principle of virtual work is not applicable for mechanical systems subjected to general nonholonomic constraints, i.e. constraints depending on the components of velocities in a general way. The aim of this contribution is to compare the physical approach to the principle of virtual work for mechanical systems with holonomic and simple nonholonomic constraints and the modern geometrical approach, which generalize the classical concept of virtual displacements and formulate the principle of virtual work for general nonholonomic constraints on the phase space.

Manifolds of higher order contact elements: examples

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The geometric structure of manifolds of contact elements is discussed. Low-dimension examples are analysed explicitly.

Global calculus of variations on spaces of mappings

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We present a global variational formulation for dynamical field theories, based on the study of critical points of functionals defined on spaces of mappings between manifolds. We will show through examples the naturality of this approach.

Differential equations with constraints in jet bundles: Lagrangian and Hamiltonian systems

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The subject of the contribution are systems of second-order ordinary and partial differential equations that arise as extremals of variational functionals in fibred manifolds. A geometric setting for EulerâLagrange and Hamilton equations, based on the concept of Lepage class is presented. A constraint is modelled in the underlying fibred manifold as a fibred submanifold endowed with a distribution (the canonical distribution). A constrained system is defined by means of a Lepage class defined on the constraint submanifold. Constrained EulerâLagrange equations and constrained Hamilton equations, and properties of the corresponding exterior differential systems, such as regularity, canonical form, or existence of a constraint Legendre transformation, are presented. This is a joint work with Olga Krupková (Palacký University, Olomouc).

RING AND MODULE THEORY

Farkas' Lemma and Linear Programming in Linearly Ordered Vector Spaces

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Firstly, we shall present the classical Farkas' Lemma, which is formulated in a real finite-dimensional vector space and is also a corner-stone for the main results of linear programming duality theory. We note that the discussed lemma can be generalized significantly: We shall present Farkas' Lemma in the setting of the vector space W, the linearly ordered vector space V, where both spaces W and V are over a common linearly ordered (possibly skew) field F. The spaces W and V play the role of the (possibly infinite-dimensional) "base" vector space and the space of "objective values", respectively. Afterwards, in the same setting, we present the generalized Duality Theorem for linear programming.

The regular algebra of a quiver

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This is a joint work with Pere Ara. Let K be a fixed field. We attach to each column-finite quiver E a von Neumann regular K-algebra $Q(E)$ in a functorial way. The algebra $Q(E)$ is a universal localization of the usual path algebra $P(E)$ associated with E. The monoid of isomorphism classes of finitely generated projective right $Q(E)$-modules is explicitly computed.

Semigroups of matrices of intermediate growth

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This is a joint work with Jan Okniński. We study the growth of finitely generated linear semigroups $S \subseteq M_n(K)$ over a field K. A general 'growth alternative' conjecture is proposed, which asserts that S has subexponential growth if and only if S has no free noncommutative subsemigroups. The problem leads in a natural way to the study of the behavior of the subexponential growth property under ideal extensions of semigroups of certain special types. This, in turn, leads to a new combinatorially defined property of groups. The conjecture is confirmed in case $n \leqslant 4$.

Two-sided localizations and bicategories

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Extending Schelter's ideas, we use the concept of bicategory to undertake a systematic study of Morita equivalences of two-sided rings of quotients. In particular we will see that maximal symmetric ring of quotients of two unital Morita equivalent rings are Morita equivalent rings. Finally an exact sequence relating Picard groups of R and its maximal symmetric ring of quotients is obtained.

The Cuntz semigroup: representations and applications to the classification program and the Blackadar-Handelman conjectures

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For a large class of unital and exact C^*-algebras, we identify the Cuntz semigroup in terms of the projection monoid and a certain semigroup of functions defined on the space of traces. This resolves two conjectures of Blackadar and Handelman and offers significant conceptual insight into Elliott's classification program. (Joint work with Nate Brown and Andrew Toms.)

Natural valuations given by uniform modules

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We show a way how a uniform module can define a valuation on the class of modules of finite Goldie dimension. Some applications including a connection to direct summands of serial modules will be given.

Representations of algebraic lattices

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We give a survey on representations of algebraic lattices as congruence lattices of various algebraic structures. First, we treat the general case, then the case of distributive algebraic lattices.

By G. Graetzer and E.T. Schmidt, every algebraic lattice is isomorphic to the congruence lattice of an algebra.

More subtle results are due to R. Freese, W. A. Lampe and W. Taylor, who proved that pinched algebraic lattices (in particular, the lattices whose maximal element is compact) are isomorphic to the congruence lattice of a monoid. However, this is not the case for the subspace lattice of an infinite dimensional vector space over an uncountable field, which is not isomorphic to the congruence lattice of an algebra of a finite similarity type.

Recently, F. Wehrung disproved the hypothesis that every algebraic distributive lattice is isomorphic to the congruence lattice of a lattice. We present various results, both on positive and negative side, related to this hypothesis. Finally we discuss representations of distributive algebraic lattices in ideal lattices of rings and partially ordered abelian groups.

Embeddings of the free algebra of inversion height one

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Let R be a domain embedded in a division ring E. It is defined inductively:

$$\begin{aligned} Q_0(R,E) &= R, \text{ and for } n \geqslant 0 \\ Q_{n+1}(R,E) &= \begin{array}{c}\text{subring of } E \\ \text{generated by}\end{array} \left\{ r, s^{-1} \mid r, s \in Q_n(R,E),\ s \neq 0 \right\}. \end{aligned}$$

Then

$$D = \bigcup_{n=0}^{\infty} Q_n(R,E)$$

is the intersection of all division subrings of E that contain R.

We define $\mathrm{h}_E(R)$, the *inversion height of* R (inside E), as ∞ if there is no $n \in \mathbb{N}$ such that $Q_n(R,E)$ is a division ring. Otherwise,

$$\mathrm{h}_E(R) = \min\{n \mid Q_n(R,E) \text{ is a division ring}\}.$$

For any division ring k and set X, we give embeddings of the free k-ring on X, $k\langle X\rangle$, of inversion height one.

Suppose $|X| = n \geqslant 2$. Consider $k[t]$, the polynomial ring with coefficients in k. Let $\alpha : k[t] \to k[t]$ be the morphism of k-rings defined by $t \mapsto t^n$. Let K be the left Ore division ring of fractions of $k[t]$. Then $k\langle X\rangle \hookrightarrow K$ by [1], and $\mathrm{h}_K(k\langle X\rangle) = 1$.

When $|X| = \infty$, we give embeddings in a similar way using what is called natural sum and product of ordinals [2].

These results are part of the joint work with Dolors Herbera.

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Ext and vanishing of derived functors of inverse limits

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The aim of this talk is to present an extension of some results of Bazzoni and Herbera. In [BH], they prove that countable inverse systems $(M_n)_{n<\omega}$ of modules with $\varprojlim^1 M_n^{(I)} = 0$ for all sets I are exactly those satisfying the Mittag-Leffler property. As a consequence, the equality $\mathrm{Ext}^1(\varinjlim C_n, X^{(I)}) = 0$ for all sets I with a countable direct system of finitely presented modules $(C_n)_{n<\omega}$ translates into the Mittag-Leffler condition on $(\mathrm{Hom}(C_n, X))_{n<\omega}$.

We will concentrate on larger inverse and direct systems and present two results for continuous systems indexed by an uncountable regular cardinal number. They, roughly speaking, replace the Mittag-Leffler property by existence of a cofinal subsystem of epimorphisms, and one of them generalizes [EM, Theorem XII.3.3] and [ST, Theorem 8]. As an application, a generalization of some results from [AH] for rings of cardinality $\leq \aleph_1$ will be presented.

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Baer modules over tame hereditary algebras

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Kaplansky problem on the structure of Baer modules over integral domains has recently been solved in [ABH]: all Baer modules are projective. The proof is by a reduction to the countable rank case (done by set-theoretic methods in [EFS]), and then by a fine analysis of countable direct limits and the Mittag-Leffler condition (in [BH] and [ABH]).

In [R], Ringel discovered close analogies between modules over Dedekind domains and over tame hereditary algebras. Following this approach, Lukas [L] and Okoh [O] studied Baer modules over tame hereditary algebras.

In my talk, I will present first results of an ongoing project with L. Angeleri and D. Herbera on the structure of infinite dimensional Baer modules. In [AT], we combine new set-theoretic methods from [ST] with the results of [O] on countable rank Baer modules in order to prove an analog of projectivity in the tame hereditary case: all Baer modules are filtered by finitely generated preprojectives. Then we recover the main result of [L] on countable rank Baer modules by an application of infinite-dimensional tilting theory. Finally, we prove a result indicating the complexity of the tame hereditary case: equivalence classes of Baer modules correspond 1-1 to isomorphism classes of all preprojectives.

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YOUNG RESEARCHERS SESSION

A dynamical systems approach to Solar Sailing

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Solar sails are a proposed form of spacecraft propulsion using large membrane mirrors. The radiation pressure on the mirror provides a small amount of thrust by reflecting photons. On the other hand, this thrust acts continuously and does not require fuel.

In this talk we will first introduce the basics of Solar Sailing. Then we will discuss a new technique to keep a spacecraft in the vicinity of an unstable fixed point. Our method is based on the computation of the effect that the orientation of the sail has on the structure of the phase space. Finally, we will show how this information can be used to design an effective control strategy.

Models of Abstract Logics acted on by a Monoid

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We extend some results of J. M. Font and R. Jansana to abstract logics acted on by a monoid. We introduce the notions of a model, a full model, the Tarski congruence,... of such structures. The principal result we will show is the existence of an isomorphism between the lattice of full models of an abstract logic acted on by a monoid and the lattice of its congruences which render the quotient a reduced model.

Advanced texture sampling

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This work deals with smooth and rough (BTF - Bidirectional Texture Function) texture modelling. Particularly BTF textures are very actual, because realistically represent appearance of given material surfaces and represent the best known representation of real object virtual models. One contribution of this work is optimization of selected sampling method for texture synthesis. Main contribution is the novel texture synthesis method for synthesizing color smooth or BTF textures.

Proposed method, which we call the Roller, is one of the intelligent sampling approaches and the method is based on the overlapping tiling and subsequent minimal error boundary cut. One or several optimal double toroidal patches or BTF patches are seamlessly repeated during the synthesis step. Due to texture analysis via Fourier transform the method is fully automatic. Moreover the method is extremely fast due to complete separation of the analytical step of the algorithm from the texture synthesis part.

The method is universal and easily implementable in a graphical hardware for purpose of real-time rendering of any type of static textures. The method also can be used for enlargement of parametric spaces indexed on regular planar grid.

Axisymmetric flow of a viscous newtonian fluid

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We study properties of solutions to the non-stationary Navier-Stokes equations in the case of axisymmetric flow. We devote ourselves to the flow in the whole $\mathbb{R}^3$ and the case of $v_\varphi \neq 0$. It is known that if v_r satisfies the so-called Prodi-Serrin condition then the solution is smooth. However, it is not known whether the same holds for v_φ where the criteria are not optimal. We improve the conditions for t, s so that for $v_\varphi \in L^{t,s}(\Omega_T)$ the solution is smooth. However, we still dont achieve optimal conditions and therefore this problem remains open.

Crossings in topological graphs

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A *topological graph* $T = (V(T), E(T))$ is a graph drawn in the plane with vertices represented as points and edges represented as simple curves connecting the corresponding pairs of points, such that the edges do not pass through any vertices other than their end-points, any two edges have only finitely many common points, any intersection point of two edges is either a common end-point or a *crossing* (a point where the two edges properly cross), and no three edges pass through a single crossing. A topological graph is called *simple* if any two edges have at most one common point.

We prove that in every simple complete topological graph with n vertices there exists an edge that crosses at most $O(n^2/\log^{1/4} n)$ other edges and we show some other related results. We also give asymptotic lower and upper bounds on the number of simple complete topological graphs on n vertices, using two different notions of isomorphism.

An *abstract topological graph* (briefly an *AT-graph*) is a pair $A = (G, R)$, where $G = (V, E)$ is a graph and $R \subseteq \binom{E}{2}$ is a set of pairs of its edges. A is *simply realizable*, if G can be drawn as a simple topological graph, where exactly the pairs of edges from R cross. We present a polynomial recognition algorithm for the class of simply realizable complete AT-graphs. On the other hand, other similar "realizability" problems are NP-hard.

The algebraic approach to Fuzzy Logic

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According to the Zadeh's famous distinction (in [7]), Fuzzy Logic in narrow sense, as opposed to Fuzzy Logic in wide sense, is the study of logical systems aiming at a formalization of approximate reasoning. In the usual semantical interpretation of these logical systems the strong conjunction connective is interpreted as a triangular norm (t-norm, for short) while the implication connective is interpreted as its residuum.

The necessary and sufficient condition for a t-norm to have a residuum is the left-continuity. In order to define the basic t-norm based fuzzy logic, Esteva and Godo introduced the system MTL in [3], which was indeed proved to be complete with respect to the semantics given by all left-continuous t-norms and their residua in [4]. Therefore, MTL is the weakest t-norm based fuzzy logic. Moreover, it can also be seen as a substructural logic since it is an axiomatic extension of the FL_{ew} calculus (see [6]) or, equivalently, of the Intuitionistic propositional calculus without contraction (see [1]), and, since Adillon and Verdú proved that the latter logic is algebraizable in the sense of [2], MTL is also algebraizable. Its algebraic counterpart is the class of the so-called MTL-algebras, already introduced in [3]. Hence, MTL it is not only a fuzzy and substructural logic, but also an algebraizable many-valued logic.

The algebraizability of a logic yields a very strong link between the logic and its algebraic counterpart, in such a way that a number of relevant logical properties correspond to algebraic properties of its semantics. Therefore, the algebraization turns out to be a powerful tool to study a logic. In the case of MTL, the algebraic approach has been a topic of interest for many mathematic logicians. In particular, it has been the topic of the dissertation [5] of the speaker. The talk will present the main ideas that are involved in this approach.

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Isotonic Regression in Sobolev Spaces

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We propose a class of nonparametric estimators for the regression models based on least squares over the sets of sufficiently smooth functions. Least squares permit the imposition of additional constraint—isotonia—on nonparametric regression estimation and testing of this constraint.

The estimation takes place over the balls of functions which are elements of a suitable Sobolev space—special types of Hilbert spaces that facilitate calculation of the least squares projection. The Hilbertness is allowing us to take projections and hence to decompose spaces into mutually orthogonal complements. We change the proof of very important Representors in Sobolev Space Theorem from [1]. We are examining representors properties and prove a theorem, which provides the way of construction of the representors and their exact form.

Then we transform the problem of searching for the best fitting function in an infinite dimensional space into a finite dimensional optimization problem—quadratic optimization with quadratic and linear constraints. We generalize this theorem into the weighted regression case. Then we investigate the form of the regression estimator in the Sobolev space. Hence we also prove symmetry and positive definiteness of the representor matrix. We use Schur Decomposition Theorem to solve our optimizing problem and prove the existence and the uniqueness of its solution. We prove the existence of 11 mapping between the Sobolev bound and the smoothing parameter.

We prove that the balls of functions in Sobolev space are bounded and have bounded higher order derivatives. It permits us to estimate over rich set of functions with sufficiently low metric entropy and apply Laws of Large Numbers and Central Limit Theorems. We also declare two main attitudes to isotonia—definite and indefinite. Finally, we implement asymptotic behavior of the regression estimator and bootstrap techniques into the algorithms.

Keywords: Isotonic regression, Sobolev spaces, nonparametric, monotonicity.

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Simplicial enrichment of model categories

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Quillen model categories provide a suitable abstract environment to do homotopy theory. For this purpose, it is often necessary to have an additional structure on the sets of mappings between two objects. We will explain how to define a simplicial enrichment of Quillen model categories, following a method due to Dwyer and Kan, and we will discuss how to solve the set-theoretical difficulties that are inherent to this method.

Counting polygon dissections in the projective plane

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For each value $k \geq 2$, we determine the number p_n of ways of dissecting a polygon in the projective plane into n subpolygons with $k+1$ sides for each. In particular, if $k = 2$ we recover a result of Edelman and Reiner (1997) on the number of triangulations of the Moebius band having n labelled point on its boundary. We also solve the problem when the polygon is dissected into subpolygons of arbitrary size. In each case, the associated generating function $\sum p_n z^n$ is a rational function in z and the corresponding generating function of plane polygon dissections. We also present some related problems.

Joint work with Marc Noy.

Telescope conjecture and cotorsion pairs of finite type

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In their paper *Application of Cotorsion Pairs*, Krause and Solberg reformulated the particular version of so-called Telescope Conjecture in the language of cotorsion pairs. The original Telescope Conjecture, stated in the late 1970s by Bousfield and lately disproved by Keller, claims that every smashing localizing subcategory of a compactly generated triangulated category $\mathcal{T}$ is of finite type. However, it is still open whether it is true when $\mathcal{T}$ is the stable module category of a self-injective artin algebra R (which is the case Krause and Solberg were interested in). In my talk, I shall present two recent results on this topic achieved, in quite a general framework, with Lidia Angeleri-Hügel and Jan Trlifaj.